Preface

The theory of differentiable manifolds and Lie groups is a basis of the study of differential geometry and differential topology, and the recent development in various branches of mathematics shows that this theory is one of the cornerstones of the edifice of modern mathematics.

The intention of this book is to provide an introduction to the theory of differential manifolds and Lie groups. The book is designed as an advanced undergraduate course or an introductory graduate course and assumes a knowledge of the elements of algebra (vector spaces, groups), point set topology, and some amount of basic analysis.

This book arose from courses given by the author at Osaka University for senior undergraduate students and is a translation of a text published in Japanese in 1965 by Shokabo, Tokyo.

The basic materials (vector spaces, topological spaces, functions of several variables) which are indispensable for the rigorous understanding of the book are gathered in Chapter I. Besides these materials we assume a knowledge of the elements of function theory of one complex variable for the understanding of few sections concerning the complex manifolds and the complex differential forms. However the reader who is not familiar with complex analysis may skip these sections.

I would like to thank Professor H. Ozeki and Professor K. Okamoto of Osaka University who have read the Japanese manuscript and given me various valuable comments.

I am indebted to late Professor E. Kobayashi of New Mexico State University for translating this book and for his contribution to the various improvements of the original text. Professor E. Kobayashi deceased just after completing the translation and I would like to dedicate this volume to his memory.

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Preface For The 2019 Edition:

As in all disciplines, the development of differential geometry is tortuous. The basic notion is that of a manifold. This is a space whose coordinates are defined up to some transformation and have no intrinsic meaning. The notion is original, bold, and powerful. Naturally, it took some time for the concept to be absorbed and the technology to be developed…..it took Einstein seven years to pass from his special relativity in 1908 to his general relativity in 1915. He explained the long delay in the following words: "Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that coordinates must have an immediate metrical meaning."

- Shiing-Shen Chern, from his Forward to *Differential Geometry:Cartan's Generalization of Klein's Erlangen Program* by R.W. Sharpe, Springer-Verlag, 1991

I originally intended for this to be a relatively brief preface. That's unusual for me, I know-I love hear the sound of my own voice. Frankly, I still believe that I should let Matshushima's beautiful text, which I have the pride and pleasure of republishing here for Blue Collar Scholar, speak for itself.[1] But

[1] I originally planned to assume anyone reading this preface has the needed prerequisites to read this book. This should be a reasonable assumption for anyone who's spent money on it. However, when was a student, I frequently dipped into considerably more advanced textbooks then I was prepared for and purchased them for future use. In respect for these daring students, I've attempted to write this preface so it will require only a

it's subject matter- differentiable or smooth manifolds-is of such immense importance to anyone serious about pursuing a career in either mathematics or the physical sciences or teaching this critical subject that I felt compelled to write an introduction for the novice about this remarkable field and its enormous importance.

If I learned anything from my undergraduate geometry courses, it was why most mathematics graduate students-both pure and applied-want to grow up to be differential geometers.

Differential geometry-both classical and modern-is the strange hybrid child of analysis, algebra and topology. It's focus is how their interplay can generate geometric structures on a specific kind of space called a manifold. The familiar Euclidean spaces of calculus and geometry,as well as the curves and surfaces embedded within them, are not only the prototypes of manifolds, but the background foundation upon which the deep structures of abstract manifolds are derived. From this interplay,a beautiful synthesis emerges that not only allows us to generalize differentiation and integration beyond the familiar curves and surfaces of R^n where $n \leq 3$, the study of the objects of analytic and synthetic Euclidean geometry is similarly generalized. It's also always been been one of my favorite subjects.

It's always seemed to me an incredibly intuitive subject — especially the classical version in Euclidean space, which focuses on curves and surfaces. I first studied classical differential geometry out of Do Carmo's *Differential Geometry of Curves and Surfaces* and the 2nd edition of O'Neill's *Elementary Differential Geometry*. Armed with basic calculus, and linear algebra in your toolbox, it seemed to me if you were

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careful understanding of multivariable calculus-as a basic real analysis course might provide-and linear algebra to read. If I failed in this aim, please let me know.